Fill in the following identities. **SCORE: _____ / 14 PTS**

[a] POWER REDUCING IDENTITY:

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

[b] HALF ANGLE IDENTITY:

$\cos\frac{1}{2}x = \pm\sqrt{\dfrac{1+\cos x}{2}}$

[c] PYTHAGOREAN IDENTITY:

$\tan^2 x = \sec^2 x - 1$

[d] NEGATIVE ANGLE IDENTITY:

$\sec(-x) = \sec x$

[e] DIFFERENCE OF ANGLES IDENTITY:

$\tan(x-y) = \dfrac{\tan x - \tan y}{1+\tan x \tan y}$

[f] SUM OF ANGLES IDENTITY:

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

[g] DOUBLE ANGLE IDENTITY: $\cos 2x = \cos^2 x - \sin^2 x,\ 2\cos^2 x - 1,\ 1 - 2\sin^2 x$

WRITE ALL 3 VERSIONS

If $\sin x = -\frac{\sqrt{7}}{4}$ and $\pi < x < \frac{3\pi}{2}$, find the values of the following expressions. **SCORE: _____ / 30 PTS**

Write each final answer as a single fraction in simplest form, including rationalizing the denominator.

[a] $\tan\frac{1}{2}x = \dfrac{\sin x}{1+\cos x}$

$= \dfrac{-\frac{\sqrt{7}}{4}}{1+-\frac{3}{4}} \cdot \dfrac{4}{4}$

$= -\sqrt{7}$

[b] $\sin(\underbrace{\arctan(-\tfrac{3}{4})}_{t} - x)$

$= \sin t \cos x - \cos t \sin x$

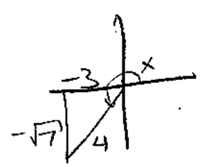
$= -\frac{3}{5} \cdot -\frac{3}{4} - \frac{4}{5} \cdot -\frac{\sqrt{7}}{4}$

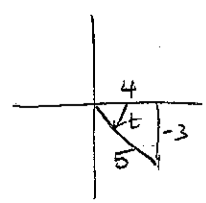
$= \dfrac{9+4\sqrt{7}}{20}$

[c] $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$= \dfrac{2\left(\frac{\sqrt{7}}{3}\right)}{1-\left(\frac{\sqrt{7}}{3}\right)^2}$

$= \dfrac{\frac{2\sqrt{7}}{3}}{\frac{2}{9}} = 3\sqrt{7}$





Prove the identity $\sec(-t)-\cos(-t)-\csc(-t)+\sin(-t)+\sin t\tan(-t)=\cos t\cot t$. **SCORE: _____ / 14 PTS**

$$= \sec t - \cos t + \csc t - \sin t - \sin t \tan t$$

$$= \frac{1}{\cos t} - \cos t + \frac{1}{\sin t} - \sin t - \sin t \frac{\sin t}{\cos t}$$

$$= \frac{1-\cos^2 t}{\cos t} + \frac{1-\sin^2 t}{\sin t} - \frac{\sin^2 t}{\cos t}$$

$$= \cancel{\frac{\sin^2 t}{\cos t}} + \frac{\cos^2 t}{\sin t} - \cancel{\frac{\sin^2 t}{\cos t}}$$

$$= \cos t \frac{\cos t}{\sin t} = \cos t \cot t$$

Rewrite $\cos^4 x$ using only the first powers of cosine (and constants and the 4 basic arithmetic operations). **SCORE: _____ / 14 PTS**
Simplify your final answer, which must **NOT** be in factored form, and must **NOT** involve any other trigonometric functions.

$$\cos^4 x = (\cos^2 x)^2$$

$$= \left(\frac{1+\cos 2x}{2}\right)^2$$

$$= \frac{1+2\cos 2x+\cos^2 2x}{4}$$

$$= \frac{1+2\cos 2x+\frac{1+\cos 4x}{2}}{4} \cdot \frac{2}{2}$$

$$= \frac{2+4\cos 2x+1+\cos 4x}{8}$$

$$= \frac{3+4\cos 2x+\cos 4x}{8}$$

Solve the equation $6-3\cos\frac{1}{5}x=5(1-\cos\frac{1}{5}x)$. **SCORE: _____ / 14 PTS**

$$6-3\cos\tfrac{1}{5}x = 5-5\cos\tfrac{1}{5}x$$

$$1 = -2\cos\tfrac{1}{5}x$$

$$\cos\tfrac{1}{5}x = -\tfrac{1}{2}$$

$$\tfrac{1}{5}x = \frac{2\pi}{3}+2n\pi \text{ or } \frac{4\pi}{3}+2n\pi, \quad n\in\mathbb{Z}$$

$$x = \frac{10\pi}{3}+10n\pi \text{ or } \frac{20\pi}{3}+10n\pi, \quad n\in\mathbb{Z}$$

Solve the equation $3\cos 2x + 7 = 7(1 - \sin x)$ algebraically. **SCORE: _____ / 14 PTS**

$$3(1-2\sin^2 x)+7 = 7-7\sin x$$

$$0 = 6\sin^2 x - 7\sin x - 3$$

$$= (3\sin x+1)(2\sin x-3)$$

$$\sin x = -\tfrac{1}{3} \quad \text{OR} \quad \cancel{\tfrac{3}{2}}$$

$$\text{REF ANGLE} = \sin^{-1}\tfrac{1}{3} \approx 0.3398$$

$$x \text{ IN } Q_3, Q_4 \qquad x = \pi + 0.3398 + 2n\pi \approx 3.4814 + 2n\pi$$

OR

$$x = 2\pi - 0.3398 + 2n\pi \approx 5.9433 + 2n\pi$$